Relativistic Mean–Field Calculations of Λ and Σ Hypernuclei *

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Abstract

Single–particle spectra of Λ and Σ hypernuclei are calculated within a relativistic mean–field theory. The hyperon couplings used are compatible with the Λ binding in saturated nuclear matter, neutron-star masses and experimental data on Λ levels in hypernuclei. Special attention is devoted to the spin-orbit potential for the hyperons and the influence of the ρ -meson field (isospin dependent interaction).

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Nowadays the hypernuclear physics are of great interest in many branches of physics. Of particular importance is the understanding of strange particles in baryonic matter, since many questions in heavy-ion physics and astrophysics are related to the effect of strangeness in matter. Experimentally and theoretically such problems have been mainly studied for the Λ hyperon, because it has zero isospin and charge. For Λ particles theoretical approaches for the spectroscopy range from nonrelativistic models [1] to the relativistic Hartree approximation (RHA) [2, 3]. For the RHA one uses a Lagrangian with effective Λ couplings to the σ - and ω -meson fields. In Ref. [3] the Λ couplings constants (i.e. their relative strength to the corresponding nucleon couplings $x_{\sigma} = g_{\Lambda\sigma}/g_{\sigma}$ and $x_{\omega} = g_{\Lambda\omega}/g_{\omega}$) have been fitted to the experimental Λ hypernuclei spectra. However, treating x_{σ} and x_{ω} as independent parameters leads to a highly uncertain determination (correlation errors up to $\pm 65\%$ in Ref. [3]).

On the other hand, the contribution of the hyperons strongly influences the mass of neutron-stars. In a recent publication [4] Glendenning and Moszkowski related the scalar and vector couplings of the Λ hyperon to its empirical binding in saturated nuclear matter [1] and thereby obtained compatibility of this binding energy with maximum neutron-star masses. In fact, the large correlation found in the least squares fit mentioned above [3] reflects this relation of x_{σ} and x_{ω} to the Λ binding in nuclear matter. In summary, one finds that (1) neutron-star masses, (2) the Λ binding in saturated nuclear matter, and (3) Λ levels in hypernuclei are mutually compatible and rather narrowly constraining the Λ couplings.

Concerning Σ hypernuclei, up to now the experimental situation is not satisfactory. Σ hypernuclear production has been investigated at CERN, and later at Brookhaven and KEK, but the statistical accuracy of the available data is not very good, because of the strong conversion the Σ undergoes in the nucleus ($\Sigma N \to \Lambda N$). The controversial evidence for narrow Σ states ($\Gamma < 5-10$ MeV) is reviewed in Ref. [5]. Therefore, in theoretical investigations

including Σ hyperons one usually assumes an universal hyperon coupling; i.e. all hyperons in the lowest octet have the same coupling as the Λ [4, 6]. The prospects for significant advances in high resolution hypernuclear spectroscopy at CEBAF or at future facilities such as the proposed PILAC and KAON are discussed in Ref. [7].

It is the aim of this contribution to analyze Λ hypernuclei under consideration of the constraints (1)–(3) mentioned above and to extend such an investigation to Σ hypernuclei.

For the nucleonic sector we use the common nuclear field theory Lagrangian including the nucleon couplings to the σ -, ω -, and ρ -meson fields [8] plus phenomenological σ -selfinteractions [9]. For the three charge states of the Σ hyperon we write the following Lagrangian [10, 11]:

$$\mathcal{L} = \sum_{\Sigma} \bar{\psi}_{\Sigma} \left(i \gamma_{\mu} \partial^{\mu} - M_{\Sigma} + g_{\Sigma \sigma} \sigma - g_{\Sigma \omega} \gamma_{\mu} \omega^{\mu} \right) \psi_{\Sigma}$$
$$- \bar{\Sigma}_{ij} \left(\frac{g_{\Sigma \rho}}{2} \gamma_{\mu} \Theta^{\mu}_{jk} + \frac{e}{2} \gamma_{\mu} A^{\mu} (\tau_{3})_{jk} \right) \Sigma_{ki}, \tag{1}$$

where Σ_{ij} and Θ^{μ}_{ij} are the traceless 2×2 matrices

$$\Sigma_{ij} = \begin{pmatrix} \psi_{\Sigma^0} & \sqrt{2}\psi_{\Sigma^+} \\ \sqrt{2}\psi_{\Sigma^-} & -\psi_{\Sigma^0} \end{pmatrix}, \tag{2}$$

and

$$\Theta_{ij}^{\mu} = \begin{pmatrix} \rho_0^{\mu} & \sqrt{2}\rho_+^{\mu} \\ \sqrt{2}\rho_-^{\mu} & -\rho_0^{\mu} \end{pmatrix}. \tag{3}$$

The sum on Σ in the first line of Eq. (1) is over the charge states Σ^- , Σ^0 , and Σ^+ . The Euler-Lagrange equations then yield the Dirac equations for the Σ hyperons:

$$(i\gamma_{\mu}\partial^{\mu} - M_{\Sigma}^* - g_{\Sigma\omega}\gamma_{\mu}\omega^{\mu} - I_{3\Sigma}g_{\Sigma\rho}\gamma_{\mu}\rho_0^{\mu} - I_{3\Sigma}e\gamma_{\mu}A^{\mu})\psi_{\Sigma} = 0, \tag{4}$$

$$M_{\Sigma}^* = M_{\Sigma} - g_{\Sigma\sigma}\sigma,\tag{5}$$

where $I_{3\Sigma}$ denotes the third isospin component; i.e. $I_{3\Sigma} = -1, 0, +1$ for Σ^- , Σ^0 , and Σ^+ , respectively. This means, that Eq. (4) already considers the fact, that within a RHA only the charge neutral component of the ρ -meson field, ρ_0^{μ} , yields a nonzero ground-state expectation value.

The Λ hyperon has isospin and charge zero and therefore cannot couple to the ρ -meson and electromagnetic fields. Hence, under the consideration of the universal hyperon coupling and the replacement of M_{Σ} by M_{Λ} in Eq. (5), the Λ Dirac equation equals the Σ^0 Dirac equation.

For calculating the hypernuclear spectra we made use of a technique similar to the so-called expectation-value method, which was successfully used within nonrelativistic and relativistic nuclear physics to incorporate shell effects into semiclassical densities and energies (see Refs. [12–14]): The Dirac-Hartree equations for the hyperons are solved only once with the meson fields of the corresponding nucleonic system, which are self-consistently determined within a relativistic Thomas-Fermi (RTF) approximation, as an input.

To check the validity of this approximation we recalculated the Λ single-particle spectra for the hypernuclei $^{40}_{\Lambda}$ Ca and $^{208}_{\Lambda}$ Pb with the parameters of Ref. [3] (parameter set I in Table I). The results for various Λ levels are displayed in Table II where our results are denoted by H^* . Compared with the fully self-consistent RHA results the H^* approximation systematically underestimates the Λ bindings which may be attributed to a surface energy that is somewhat too large within the RTF approach [15]. But as expected, the agreement is better for the larger mass number A and the deeper lying levels because in both cases the Thomas-Fermi assumption of locally constant fields is more valid. In conclusion, our results show a rather good agreement with those of Ref. [3], which gives confidence in the described scheme.

In the next step we calculated several Λ and Σ hypernuclei using a set of coupling constants from Ref. [4], which considers the constraints (1)–(3) mentioned above and in addition has been successfully used in the description of nuclear matter properties (parameter set II in Table I). In Fig.1 we show the contributions of the meson and electromagnetic fields to the hyperon self-energy for the nuclei ²⁸Si, ⁴⁰Ca, ⁹⁰Zr, and ²⁰⁸Pb. The nonrelativistic reduction of the hyperon potential, (for Λ and Σ^0 entirely, for Σ^{\pm} mainly) given by the difference $g_{H\omega}\omega^0 - g_{H\sigma}\sigma$, $H = \Lambda, \Sigma$, is also displayed. It is worth noting, that the small potential depths of ~ 30 MeV go along with a relatively smooth radial dependence (compared with nucleonic potentials), thereby additionally supporting the feasibility of the Thomas-Fermi meson fields. A similar behaviour was found for $^{16}_{\Lambda}O$ within the RHA calculations of Ref. [3]. In Figs.2 and 3 we show the single-particle spectra of protons, neutrons, Λ , Σ^0 , and Σ^+ hyperons for the nuclei ⁴⁰Ca and ²⁰⁸Pb. Because of the smaller couplings the hyperon levels are considerably less bound than

the corresponding nucleon levels. Looking at the Λ and Σ^0 single-particle energies, the larger M_{Σ} yields a smaller repulsive effect of the kinetic energy resulting in systematically stronger bindings for the Σ^0 .

Dealing with hypernuclear states and their structure, one of the most interesting questions concerns the spin-orbit potential for the hyperons [7]. It is one of the great advantages of a relativistic treatment that the spin-orbit interaction is automatically included in the single-particle Dirac equation, and can be identified by means of a Foldy-Wouthuysen reduction. For example, looking at the charge neutral Λ and Σ^0 hyperons, both of which are not coupled to the ρ -meson field, the ratio of the spin-orbit splitting (Thomas terms) is

$$\frac{V_{s.o.}^{\Sigma^0}}{V_{s.o.}^{\Lambda}} = \frac{M_{\Lambda}^2}{M_{\Sigma}^2} = 0.88 . {(6)}$$

This ratio is very well reproduced in the corresponding spectra of Figs. 2 and 3. Concerning the ratios of the spin-orbit splitting of the proton and neutron to the Σ^+ and Σ^0 hyperon, respectively, a simple expression similar to the one of Eq. (6) cannot be given because of the different couplings. However, in the corresponding spectra of Figs. 2 and 3 we found a value of about ~ 0.34 .

In Table III we compare various Σ^- , Σ^0 , and Σ^+ levels for the ${}_{\Sigma}^{28}\mathrm{Si}$, ${}_{\Sigma}^{40}\mathrm{Ca}$, $_{\Sigma}^{90}$ Zr, and $_{\Sigma}^{208}$ Pb hypernuclei. Of course, the Coulomb force plays an important role: comparing Σ^- with Σ^0 levels, the atomic states disappear, whereas for the Σ^+ states the baryonic potential has to overcome the Coulomb repulsion with the effect that only the deep lying states survive. For symmetric (N=Z) hypernuclei, where the ρ -meson field is weak (it is nonzero because the proton and neutron density distributions differ due to the Coulomb interaction), we found the Coulomb shifts between Σ^- and Σ^0 , or Σ^0 and Σ^+ states almost identical to the corresponding neutron-proton shifts in "normal" symmetric nuclei. The situation is somewhat different for hypernuclei with a neutron-excess because then the effect of the ρ -meson field is not negligible anymore: For the Σ^- (Σ^+) the ρ -meson adds (subtracts) from the isoscalar part of the timelike repulsive vector field. The Σ^0 does not couple to the ρ field at all. To get an idea of the impact of the ρ -meson field we recalculated the asymmetric hypernuclei ${}^{90}_{\Sigma}{\rm Zr}$ and ${}^{208}_{\Sigma}{\rm Pb}$ with the ρ - Σ coupling switched off. For Σ^{90} Zr we found the $\Sigma^{-}(\Sigma^{+})$ states stronger (weaker) bound by about 2.1–3.5 MeV; the corresponding range for Σ^{208} Pb is 4.4–6.9 MeV. The fact that such relatively large ranges occur can be understood in terms of the rms-radii: The lower bounds are for weakly bound states with large rms-radii (e.g. the $1g_{9/2}$ Σ^- state with $r_{rms} = 5.40$ fm for ${}_{\Sigma}^{90}$ Zr and the $3p_{1/2}$ Σ^- state with $r_{rms} = 6.97$ fm for ${}_{\Sigma}^{208}$ Pb), while the upper bounds correspond to deep lying states with small rms-radii (e.g. the $1s_{1/2}$ Σ^- states with $r_{rms} = 3.09$ fm for ${}_{\Sigma}^{90}$ Zr and $r_{rms} = 3.68$ fm for ${}_{\Sigma}^{208}$ Pb; the values for the rms-radii are from the calculations with the ρ field switched on). As one can see, the influence of the ρ -meson field, whose range is determined by its mass m_{ρ} , weakens with increasing radial distances. (For neutrons and protons the situation is similar to the $\Sigma^- - \Sigma^+$ pair but the effective ρ -nucleon coupling is weaker.)

Hence, the quantum hadrodynamical treatment of hypernuclei offers, by the possible inclusion of the ρ -meson field, a natural way to incorporate an isospin dependence into the Σ potential (i.e. a Lane potential), which was pointed out by Dover in Ref. [7] to be one of the most important questions of hypernuclear physics.

Finally, we show in Figs.4–6 the single-particle energies of the Σ^- , Σ^0 , and Σ^+ hyperons, respectively, versus $A^{-2/3}$, with, A the mass number of the nuclei. For the Σ^- (Fig.4) the attractive Coulomb potential alone, irrespective of the strength of the short-range nuclear potential, is enough to bind Σ^- states. Some of these states (the most bound) are such that the rms-radii of the Σ^- wavefunctions are essentially inside the nucleus: Looking at $^{40}_{\Sigma^-}$ Ca we found for the rms-radii of the plotted s_{Σ^-} , p_{Σ^-} , d_{Σ^-} , and f_{Σ^-} states the values of $r_{rms}=2.65, 3.31, 3.89,$ and 4.76 fm, respectively. Therefore, compared with the experimantal charge rms-radius of $r_c \sim 3.48$ fm for 40 Ca, the d_{Σ^-} and f_{Σ^-} states should be called atomic ones, while for the other levels an identification as hypernuclear states seems to be more appropriate.

For the charge neutral Σ^0 (Fig.5) there is no Coulomb attraction and therefore the number of bound states decreases. The value of -28 MeV for $A^{-2/3} = 0.0$ represents the binding energy of the lowest Σ^0 level in nuclear matter, which is under the assumption of an universal hyperon coupling the same as for the Λ particle [1, 4]. As expected, the pattern of states shows the standard behaviour as for Λ hypernuclei [3].

Turning finally to the discussion of the A-dependence of the Σ^+ levels (Fig.6) the situation becomes more complicated. Of course, now the Coulomb force is repulsive and the number of bound states further decreases compared with Σ^- and Σ^0 hypernuclei. But to get a full understanding of Fig.6, it

seems necessary to consider the various contributions to the nuclear potential in view of their range, which is determined by the corresponding meson mass, respectively. For example, the fact that the binding of the d_{Σ^+} state increases going from A=90 to A=208, while there is an opposite effect for the s_{Σ^+} and p_{Σ^+} states, may be attributed to the larger d_{Σ^+} rms-radius (e.g. 5.63 fm instead of 4.55 fm and 5.17 fm for the s_{Σ^+} and p_{Σ^+} levels in 208 Pb, respectively). The d_{Σ^+} wavefunction is located at large radial distance, where the impact of the attractive σ -meson ($m_{\sigma}=500$ MeV) increases locally compared with the repulsive ω -meson ($m_{\omega}=783$ MeV) due to their different ranges. Seemingly, the various states show a behaviour similar to the one found within nonrelativistic Hartree-Fock Skyrme calculations for protons in "normal" nuclei, where the binding of the d levels strongly increases, whereas the s states nearly stay constant when going from A=90 to A=208 [16].

In the present calculations the broadening of the Σ hyperon states due to their decay to the Λ was neglected. In principle, the model can be extended to include the decay by introducing appropriate $\Sigma\Lambda$ -vertices. An important aspect of such an extended approach would be the possibility of investigating the decay of strange particles in the nuclear medium. In order to estimate the effects due to the conversion $\Sigma N \to \Lambda N$ the results of nonrelativistic potential models [17] may be taken as a guideline. In such approaches the decay is described schematically by an absorption potential for hyperons. The imaginary part of the self-energy effectively lowers the binding energy which can be understood in terms of the pole structure of the baryon propagator. Qualitatively a similar effect has to be expected also in a covariant description including the decay of the Σ hyperons. Thus the present results are likely to give lower bounds for the binding properties of the Σ particles in hypernuclei.

In conclusion, we performed relativistic mean-field calculations of Λ and Σ hypernuclei using an interaction that considers neutron-star masses, the Λ binding in saturated nuclear matter, and experimental Λ single-particle levels. Concerning the Σ couplings we assumed an universal hyperon coupling; i.e. all hyperons in the lowest octet couple to the meson fields as the Λ . We employed the so-called expectation-value method whose reliability was found to be sufficient compared with fully self-consistent RHA calculations. Analyzing the hypernuclear spectra, we devoted special attention to the spin-orbit interaction for hyperons and, in the case of Σ hypernuclei, to the isospin dependence of the interaction. These two features are of particular interest in the current discussion concerning hyperon potentials in nuclei and are nat-

urally incorporated into the relativistic quantum hadrodynamical model we used. In addition we found by comparison with corresponding nonrelativistic results that for strongly bound states the impact of the $\Sigma N \to \Lambda N$ decay width on the bindings seems to be negligible.

In the future it would be very valuable from the point of view of dense matter properties, and especially the structure of neutron-stars, to have the assumption of an universal hyperon coupling confirmed by detailed precision experiments on Σ hypernuclei, and we hope that these calculations may possibly be of assistence as well as a stimulus to such experiments and the development of the necessary facilities.

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Table captions

- table I: Parameters of the two forces considered in the text. In both cases the nucleon mass M=938 MeV. For saturated nuclear matter the set II [4] yields energy per particle E/A=-16.3 MeV, density $\rho_0=0.153$ fm⁻³, incompressibility K=300 MeV, effective mass $M^*/M=0.7$ and symmetry energy coefficient $a_{sym}=32.5$ MeV. $C_i^2=g_i^2\left(M/m_i\right)^2, x_i=g_{Hi}/g_i; i=\sigma,\omega,\rho; H=\Lambda,\Sigma.$
- table II: Comparison of our H* results for various Λ levels in $^{40}_{\Lambda}$ Ca and $^{208}_{\Lambda}$ Pb with the fully self-consistent relativistic Hartree results from Ref. [3]. All quantities are in MeV.
- table III: Various Σ^- , Σ^0 , and Σ^+ levels in ${}^{28}_{\Sigma}\mathrm{Si}$, ${}^{40}_{\Sigma}\mathrm{Ca}$, ${}^{90}_{\Sigma}\mathrm{Zr}$, and ${}^{208}_{\Sigma}\mathrm{Pb}$. All quantities are in MeV.

Table I

	I [3]	II [4]
$m_{\sigma} \; (\mathrm{MeV})$	499.31	500
$m_{\omega} \; (\mathrm{MeV})$	780	783
$m_{\rho} \; (\mathrm{MeV})$	763	770
$M_{\Lambda} \; (\mathrm{MeV})$	1116.08	1115
$M_{\Sigma} \text{ (MeV)}$	_	1190
C^2_{σ} C^2_{ω}	348.26	266.40
C^2_{ω}	229.29	161.53
$C^2_{ ho}$	148.92	99.67
$b \times 10^3$	2.2847	2.947
$c \times 10^3$	-2.9151	-1.070
x_{σ}	0.464	0.600
x_{ω}	0.481	0.653
x_{ρ}	_	0.600

Table II

	$^{40}_{\Lambda}{ m Ca}$		$^{208}_{\Lambda}{ m Pb}$		
Level	RHA [3]	H^*	RHA [3]	H^*	
$1d_{3/2}$	-2.63	-1.17	-15.78	-15.03	
$1d_{5/2}$	-3.76	-2.08	-16.12	-15.32	
$1p_{1/2}$	-10.93	-8.75	-20.38	-19.42	
$1p_{3/2}$	-11.61	-9.38	-20.51	-19.54	
$1s_{1/2}$	-19.43	-16.90	-24.19	-23.23	
,					

Table III

Level	Σ^-	$\overset{28}{\Sigma}\mathbf{Si} \ \overset{2}{\Sigma}^0$	Σ^+	Σ^-	$\overset{40}{\Sigma}\mathbf{Ca} \overset{50}{\Sigma}$	Σ^+
$1f_{5/2}$	_	_	_	-2.31	_	_
$1f_{7/2}$	_	_	_	-3.43	_	_
$2s_{1/2}$	-5.19	-0.43	_	-9.73	-2.57	_
$1d_{3/2}$	-5.10	_	_	-10.29	-3.00	_
$1d_{5/2}$	-6.19	-0.87	_	-11.31	-3.99	_
$1p_{1/2}$	-13.97	-7.96	-2.07	-18.79	-10.82	-3.00
$1p_{3/2}$	-14.71	-8.69	-2.79	-19.37	-11.42	-3.60
$1s_{1/2}$	-23.22	-16.63	-10.11	-27.10	-18.54	-10.07
		$^{90}_{\Sigma}{ m Zr}$			$_{\Sigma}^{208}\mathbf{Pb}$	
$1f_{5/2}$	-13.35	-4.03	_	-26.47	-12.40	_
$1f_{7/2}$	-14.30	-5.00	_	-26.87	-12.87	_
$2s_{1/2}$	-19.35	-9.28	_	-30.79	-15.76	-0.73
$1d_{3/2}$	-20.29	-10.52	-0.87	-31.57	-16.99	-2.70
$1d_{5/2}$	-20.88	-11.15	-1.53	-31.76	-17.24	-3.02
1n.	-26.90	-16.61	-6.47	-36.25	-21.06	-6.22
$1p_{1/2}$	=0.00					
$1p_{1/2} \ 1p_{3/2}$	-27.16	-16.91	-6.80	-36.31	-21.15	-6.36

Figure captions

- figure 1: Hyperon self-energy contributions $g_{H\sigma}\sigma$ (dotted lines), $g_{H\omega}\omega^0$ (dashed lines), $g_{\Sigma\rho}\rho_0^0$ (long-dashed lines), and eA^0 (dot-dashed lines) for the nuclei ²⁸Si, ⁴⁰Ca, ⁹⁰Zr, and ²⁰⁸Pb. The nonrelativistic reduction of the hyperon potential is (for Λ and Σ^0 entirely, for Σ^{\pm} mainly) given by the difference $g_{H\omega}\omega^0 g_{H\sigma}\sigma$, which is represented by the solid lines, respectively. $H = \Lambda, \Sigma$ in the hyperon coupling constants.
- **figure 2:** The calculated proton, Σ^+ , neutron, Λ , and Σ^0 single-particle spectrum for ⁴⁰Ca (parameter set II of Table I).
- figure 3: Same as Fig.2 for ²⁰⁸Pb.
- figure 4: Single-particle energies versus $A^{-2/3}$ for Σ^- . For each angular momentum the lowest lying state is plotted, respectively. The dashed lines are added to guide the eye.
- **figure 5:** Same as Fig.4 for Σ^0 . The value of -28 MeV for $A^{-2/3} = 0.0$ represents the binding energy of the lowest Σ^0 level in saturated nuclear matter under the assumption of an universal hyperon coupling [1, 4].
- **figure 6:** Same as Fig.4 for Σ^+ .



